Topic: Proving that the limit does not exist

Question: How do we know that the limit does not exist?

$$\lim_{x \to 0} \frac{1}{x}$$

Answer choices:

A The left-hand limit is zero.

B The right-hand limit does not equal the left-hand limit.

C The right-hand limit does equal the left-hand limit.

D The right-hand limit does exist.

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Solution: B

A limit exists if both left-hand and right-hand limits exist and are equal. To prove that a limit does not exist, find the right-hand and left-hand limits. If one or both does not exist, then the limit does not exist. If they both exist, check to see if they're equal to each other. If they're not equal, the limit does not exist.

Start by finding the right-hand and left-hand limits.

We'll use 2, 1 and $1/2$ in the right-hand limit.

$$\lim_{x \to 0^+} \frac{1}{x} = \frac{1}{2}$$

$$\lim_{x \to 0^+} \frac{1}{x} = \frac{1}{1} = 1$$

$$\lim_{x \to 0^+} \frac{1}{x} = \frac{1}{\frac{1}{2}} = 2$$

We can see that as x gets closer to 0 from the right-hand side, the limit is approaching ∞.

We'll use -2, -1 and $1/2$ in the left-hand limit.

$$\lim_{x \to 0^-} \frac{1}{x} = -\frac{1}{2}$$

$$\lim_{x \to 0^-} \frac{1}{x} = \frac{1}{-1} = -1$$

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$$\lim_{x \to 0^-} \frac{1}{x} = \frac{1}{-\frac{1}{2}} = -2$$

We can see that as x gets closer to 0 from the left-hand side the limit approaches $-\infty$.

So far we know that both limits exist. Next we need to check to see if the two limits equal each other. But

$$+\infty \neq -\infty$$

Since the two limits do not equal each other, the general limit does not exist.

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Topic: Proving that the limit does not exist

Question: How do we know that the limit does not exist?

$$\lim_{x \to 0} \frac{\sqrt{x^2}}{x}$$

Answer choices:

A The right-hand limit is 1.

B The right-hand limit does equal the left-hand limit.

C The right-hand limit does not equal the left-hand limit.

D The left-hand limit does exist.

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Solution: C

A limit exists if both left-hand and right-hand limits exist and are equal. To prove that a limit does not exist, find the right-hand and left-hand limits. If one or both does not exist, then the limit does not exist. If they both exist, check to see if they're equal to each other. If they're not equal, the limit does not exist.

Start by finding the right-hand and left-hand limits.

We'll use 2, 1 and $1/2$ in the right-hand limit.

$$\lim_{x\to 0^+} \frac{\sqrt{x^2}}{x} = \frac{\sqrt{4}}{2} = \frac{2}{2} = 1$$

$$\lim_{x\to 0^+} \frac{\sqrt{x^2}}{x} = \frac{\sqrt{1}}{1} = \frac{1}{1} = 1$$

$$\lim_{x\to 0^+} \frac{\sqrt{x^2}}{x} = \frac{\sqrt{\frac{1}{4}}}{\frac{1}{2}} = \frac{\frac{1}{2}}{\frac{1}{2}} = 1$$

We can see that as x gets closer to 0 from the right-hand side, the limit is approaching 1.

We'll use -2, -1 and $-1/2$ in the left-hand limit.

$$\lim_{x\to 0^-} \frac{\sqrt{x^2}}{x} = \frac{\sqrt{4}}{-2} = \frac{2}{-2} = -1$$

$$\lim_{x\to 0^-} \frac{\sqrt{x^2}}{x} = \frac{\sqrt{1}}{-1} = \frac{1}{-1} = -1$$

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$$\lim_{x\to 0^-}\frac{\sqrt{x^2}}{x}=\frac{\sqrt{\frac{1}{4}}}{-\frac{1}{2}}=\frac{\frac{1}{2}}{-\frac{1}{2}}=-1$$

We can see that as x gets closer to 0 from the left-hand side the limit approaches -1.

So far we know that both limits exist. Next we need to check to see if the two limits equal each other. But

$$1\neq -1$$

Since the two limits do not equal each other, the general limit does not exist.

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Topic: Proving that the limit does not exist

Question: How do we know that the limit does not exist?

$$\lim_{x \to 1} \ln(x - 1)$$

Answer choices:

A The right-hand limit does not exist.

B The right-hand limit does equal the left-hand limit.

C The left-hand limit is approaching $-\infty$.

D The left-hand limit does not exist.

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Solution: D

A limit exists if both left-hand and right-hand limits exist and are equal. To prove that a limit does not exist, find the right-hand and left-hand limits. If one or both does not exist, then the limit does not exist. If they both exist, check to see if they're equal to each other. If they're not equal, the limit does not exist.

Start by finding the right-hand and left-hand limits.

We'll use 3, 2 and $3/2$ in the right-hand limit.

$$\lim_{x\to 1^+} \ln(x-1) = \ln(3-1) = \ln(2) \approx 0.69$$

$$\lim_{x\to 1^+} \ln(x-1) = \ln(2-1) = \ln(1) = 0$$

$$\lim_{x\to 1^+} \ln(x-1) = \ln\left(\frac{3}{2}-1\right) = \ln\left(\frac{1}{2}\right) \approx -0.69$$

We can see that as x gets closer to 1 from the right-hand side, the limit is approaching $-\infty$.

We'll use -1, 0 and $1/2$ in the left-hand limit.

$$\lim_{x\to 1^-} \ln(x-1) = \ln(-1-1) = \ln(-2) = \text{DNE}$$

$$\lim_{x\to 1^-} \ln(x-1) = \ln(0-1) = \ln(-1) = \text{DNE}$$

$$\lim_{x\to 1^+} \ln(x-1) = \ln\left(\frac{1}{2}-1\right) = \ln\left(-\frac{1}{2}\right) = \text{DNE}$$

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We can see that as x gets closer to 1 from the left-hand side that the limit does not exist.

Because the left-hand limit does not exit, the general limit does not exist.

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